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The Turning

And now for something completely different ...

The return

I return to *Wilmott* and to “serving the quantitative finance community” after writing *The Blank Swan: The End of Probability*, or the book that has purged contingency from the metaphysics of possibility and prediction and correlatively has freed the mathematics of contingency from the mediation of probability.¹ In my book, I have proposed replacing probability with a new kind of number, which I call the *price*, and the calculus of probability with the *mathematics of price*. The market price of contingent claims, I hold, is the direct translation of contingency. As for probability, it is only a metaphysical – that is to say, an artificial – detour.

It took me over two years to write *The Blank Swan*, as well as to free myself from the whole framework of thought that still abided by the logic of probability and states of the world, also called the “replication strategy.” Not only did I have to retire from my duties of planner and organizer of the software company that first brought me to writing in *Wilmott* (ITO 33) in order to write the book (and I do thank my partners for letting me take that leave, not to mention the editors of *Wilmott*, whom I have deprived of my regular column in the interval), not only did I have to get outside the frame, the plan, the program – what, speaking of companies and their heads, is called *boîte* in French, or box – but I also had to get outside the *book*.

The book itself is but a frame or a program. Writing a book also amounts to a dynamic replication strategy. Therefore, part of the idea of fulfilling the promise of the subtitle and of getting the “end of probability” inside my book (or the end of replication, or the end of the derivative;

that is to say, the beginning of the contingent claim) was to get that thought *outside* the book altogether and outside its conceptual binding. How could contingent claims be thought absolutely, as deriving from no underlying state, no metaphysics, and no probability – how could their market become materialized in a book – and the book itself, as a binder and a list of contents or a range of fixed possibilities, not come absolutely out of joint?

Overturning probability and metaphysical thought – this revolution – cannot but extend to all the domains where the logic of state and plan and expectation rules. Therefore, the book of the “end of the probability” can only begin with the end of the book. Theory and narrative, or the two dimensions spanning the material that books are usually made of (I include fiction under the heading of theory), have to mingle their strands before their joint knot gets inverted. This invaginating movement takes place in Part III of my book.

Returning to the market and to the reality of prices of contingent claims and finding what the market is and what prices truly are can only mean, in the end, inverting the box and inverting the book. It can only mean walking back on the surface, coming from the other end of probability – exactly what market-makers do when they invert the pricing formula in order to price the contingent claims and walk on that surface.

For this reason, I can come back now to quantitative finance with the simplest idea in mind: the idea *after* the book and after the inversion. Now that the book is behind me, together with all the strings that once attached me to the ancient frames of thought (to the book, to the *boîte*, even to the *bank* as the recent representative of the failure of probabilistic thought and bad speculation), I can encapsulate the crucial distinction between contingency and possibility in a single stipulation. What is cru-

cially required, in order to distinguish between contingent claims and derivatives (or between price and probability, or between market and metaphysical thought, or between exchange place and chronological time) is that we may distinguish between the thought of the other worlds that the actual world *possibly* can be – the so-called possible worlds or states of the world – and the massive and undividable thought that the world is what it actually is, in reality and not in possibility, *except that it could have been different*. The first thought is a complete fabrication. The second is absolutely real.

Possibility and the passage of time

Possibility is intimately linked with chronological time. We talk of *temporal* processes or equivalently of stochastic processes. Chronological time introduces the idea that history forks into different possibilities facing the world and that reality selects the branch that becomes actualized. If we symbolize reality with “1” and unreality with “0,” we can observe the symbol of reality transitioning from node to node as time passes, while the branches that are not selected are annulled and drop off the tree.

Probability tries to answer the question of what number to assign to the different nodes before the march of time reaches them. By contrast, the massive thought that the actual world, as finished and settled as it may be, could have been different faces no such fork as time advances. Indeed, this thought always sounds as an afterthought, as if it wanted to take place after time has made its selection between the different possibilities. We may even say that thinking that the world could have been different is something we can experience anytime, independently of time and the fork of possibilities. Why does contingency have to be processed by time? Can’t we withdraw time altogether from the thought of contingency?

Certainly, time passes. Certainly, the world will be something real tomorrow and it will almost as certainly prompt the thought that it could have been otherwise. However, we wish to keep the temporal passage from a day to the next dark and massive. We don't wish to *mediate* it via the channel of possibility. We don't want to process it as a transition between visible states. In other words, we don't want a temporal process (stochastic, as it were) to take care of the transition. When a thing actually exists, it is easy to think that it could have been different without *identifying* the difference or the "other things" that it could have possibly been. The capacity of being different remains inherent in the thing, so to speak. It doesn't necessarily get distributed outside, among other identifiable states. There is even a compelling philosophical suggestion that contingency might be ontologically first and that, before we even think of being and of what the thing *is*, we should think of contingency and of what the thing *can* be.²

When the thing is not yet actualized, its unavailability induces us into thinking of its contingency in terms of different possibilities, one among which is the actual thing as it will come to be realized. However, Bergson teaches us that contingency is real while possibility is not.³ To think of a future contingent thing, we shouldn't, therefore, think of future possibilities. Rather, we should think that the thing *will be* real – it is in future reality, not in present possibility, that the thought of the future should take place – and that it *will be* contingent as an inherent aspect of that future reality. Chronological time makes us confuse this with possibility. Because we think of a nonactual thing that could be different, we think of it as a possibility among *others*. There is a separation of branches because of the chronological gap. (Internal difference translates into the external "others.") We think that possibility precedes reality. We confuse contingency with indeterminism. We think that the future thing is undetermined between several possible alternatives before thinking what actually it will be.

In fact, thinking that a thing *can be* (that is to say, it is contingent) must occur before thinking that it actually *is*, and the last thought, in turn, must occur before thinking what possible

states it will be in and how to vary those states. Thinking a nonactual thing should not be confused with thinking an unreal, or possible, thing. Thought can (and must) anticipate reality.

Possible state vs. real contingency

Let us deconstruct probability. I claim that probability (and what underlies it – namely, the possible states it is usually assigned to) is unreal and that the only real thing is contingency. Before we think of a fork of possibilities facing the world and of the metaphysical notion of "states of the world," before we think of probabilistic transitions and of processes taking place in time, our first massive and undivided thought should be that the world exists, that it is real and that it is actual, *only it could have been different*. This is the primordial thought of contingency. This is the raw material. By contrast, thinking of possibility is sophisticated and is adding to this thought. It involves the fiction of the other possible worlds and is a fabrication. For this reason, it comes after reality, not before. It is by virtue of a metaphysical reconstruction and sophistication (sophism?) of thought that we imagine that possibility precedes reality and that the different possibilities facing the world become realized as time passes.

The first immanent thought is that the world is just the way it is – that it is unexchangeable and that it has emerged at a stroke.⁴ Even the question of whether the world is necessary or not (the best of all possible worlds) is a later addition and an unnecessary exchange of the world. The first and underived thought (the absolute thought) is that the world is contingent without a question. To imagine a space of possibilities is already to elevate a transcendent dimension above the world and a transcendent realm in which the process of change is supposed to take place.

Time is the second immanent thought. We know that time passes. We wake up every day and the world once again is the way it is and it is still contingent. However, to link the passage of time with the realization of possibilities is a false, transcendent move. It involves putting the massive thought of contingency inside the thought of the passage of time. It involves the copula of transcendence: IS. We imagine that

time is the reason why the world changes and why it is contingent. We imagine that the future world is different from the present one. However, the copula of immanence is AND. What we have is the immanent thought of contingency *and* the immanent thought of time. As such, the two are unrelated. There is no process putting one inside the other. Any such process is a later fabrication. The world exists every day and it is contingent every day. Its contingency is repeated, so to speak. We can think of its contingency anew every day without thinking of the different worlds that it possibly can be. It is enough to think that it is not necessary.

The false transition between the possible and the real

To think that the world is not necessary is to think that it could have been different. This does not necessarily entail the distinct thought of the alternative worlds that it could have been. We are accustomed to thinking of identity first, then of difference. We first identify the world, then we identify the difference; that is to say, we identify (or think we identify) the other worlds that it possibly can be. We thus think of difference externally and analogically. The image of the possible worlds is projected after the real world. However, why first *identify* the world? Why first exercise identity upon the world? To insist on delivering identity cards is but a statist requirement. Is the world really in need of a passport? What if difference were, on the contrary, thought before identity?

That the world is the way it is – that it is contingent – is not the recognition of an identity. The stroke of contingency is faster than identification and more primitive than identity. Try the alternative ontology (Meillassoux). Try to think that the world is contingent before identifying what the world is, or even that it is.

The world is contingent. The world could have been different. This has to be thought before we think that the world exists. As a matter of fact, existence is *derived* from contingency in Meillassoux. At best, the thought of the way the world is – its actual existence – should take place at the same time as the thought of its contingency.



If the contingency of the world (the fact that it could have been different) is thought before its existence and identity in this way, then the thought of the other identifiable worlds will appear to be even more derivative and more secondary. Therefore, to be more exact, we shouldn't say that the contingency of the world is the negation of its necessity, but the opposite. Thinking of a necessary world is parasitical on contingency and is less parsimonious than thinking of a contingent world.

Every day we know that the world could have been different. Metaphysics commits the first mischief against the immanence of contingency when it imagines the different *state* of the world. This is how it first steps outside. However, two different worlds cannot actually coexist. The actuality (or reality) of the first repels the idea of the second. Therefore, the second step that metaphysics undertakes is to make the two worlds equal by thinking of their coexistence *before* the actuality of the first. This is how metaphysical thought makes use of time. It knows that the world can be different today and now it tries to keep hold of that thought and literally to "remember" it as it steps backwards to the day before. This is how possibility is fabricated. The two different worlds cannot coexist in reality, so metaphysical thought lets them coexist in unreality and the only available unreality, as far as metaphysical thought is concerned, is a future that it fabricates by artificially stepping back in the past.

Thinking of future possibilities (or future unrealities) is only the thought of the different world that the actual contingent world could have been artificially projected backwards to a time when the actual world is supposed not to be actualized yet. This false move creates the necessity of the transition. Because thought has artificially moved backward, it has to reverse its steps and move forward again; therefore, a transition has to occur and one of the two possible worlds that it has identified now has to become real or actual. The possible is staged and thought has to walk back and forth on that stage. As time really moves forward, however, nothing guarantees that one of the two imagined possibilities will be recovered. We move backward just for the

sake of making the two states of the world coexist, and we are bound to reverse our course and to move forward almost as quickly just to verify our "hypothesis." Chances are, however, that the real world has never moved backward with us in the first place. It is always ahead of us and of our thought of the different possibilities. We should have never thought of the other different worlds that the contingent world possibly can be. Probability is just the numerical translation of this fabrication.

The symbol of reality

There is only one world, but it could have been different. Possibility thinks difference analogically, after identity. It derives difference from identity and separates it from identity. It *identifies* the difference. It imagines that the possible worlds differ with respect to the so-called "states of affairs," when there is nothing to guarantee that the actual world can first be reduced to identifiable states. This is how the metaphysical thought of possibility opposes to the "1" of the real world the "0" of the other world. This is how the idea first emerges to assign a number (probability) to a world, or to a state of the world.

With probability, the world falls into external difference. It becomes a state, a fiction, when it should have remained real and massively contingent. The mathematics of possibility is articulated in saying that the world receives "1" if it is realized, "0" if it is not realized. "1" thus becomes the symbol of reality.

"1" is the symbol of reality and it should have remained the only symbol. In fact, "0" is not the symbol of unreality. Contingent reality is absolute; it is a single stroke and it cannot be contradicted because it cannot be relativized. Rather, "0" is the symbol of identity. It is the *consequence* of the analogical identification of the actual world taking precedence over the thought of its internal difference. When difference (or primordial contingency) is taken outside and thought *after* the identity of the actual world, it immediately freezes into the thought of the other states of the worlds (the different worlds that the world could have been), and those illegal worlds then receive the "0" symbol. This is why, paradoxical as it may sound, I maintain that "0" is the symbol

of identity (or identification of the actual world). To repeat, "1" should have admitted of no opponent – of no "0." It symbolizes reality (i.e., contingency), not identity and identification.

It is now too late, however, and the sole and undivided symbol of reality, "1," becomes a number thrown on a numerical scale as a consequence of its association with "0." As for the symbol of *unreality* (which is neither "0" nor "1," as we will see), it emerges after the symbol of identity and analogy, "0," has first mediated and exchanged the once unexchangeable reality of "1."

As soon as "0" emerges, it forms a couple with "1" and exchanges it. The different states of the world, which are the consequence of the identity of the actual word taking precedence over its contingency, are bound to exchange the "0" symbol they get assigned against the symbol of reality. As soon as internal difference freezes into external different states, those identifiable states now all equally claim their share of reality just in order to complete the exchange. From reality (the undivided and contingent "1") to identity (the emergence of "0") to the illegal exchange or intercourse (the couple formed by "1" and "0") to the identification of the different states and the reality they claim, it is all but one sequence leading inexorably to the thought of possibility – that is, unreality.

The symbol of unreality

The symbol of unreality is the number p which is strictly comprised between 1 and 0 and whose name is *probability*. The different worlds cannot share the symbol of reality with the real world because, by the principle of bivalence, two different worlds are incompatible in reality. Nor can they *actually* exchange their "0" symbol against the symbol of reality because the one real world is the only actual world. The impossible exchange between the real, undivided, and contingent world and its identifiable states *thus becomes the thought of possibility*. Possibility, now artificially projected backwards before the actuality of the real world, is the only place where all the worlds can be equal and exchange the symbol of reality among each other.

The thought of the symbol of reality, "1," now being exchanged and permutated between the

equally possible worlds soon gives way to the thought of a number, strictly less than 1, that the possible worlds can *simultaneously* share. Because probability is less than one, it is able to circumvent the principle of bivalence. The numerical scale extending between 0 and 1 is precisely available in order to shelter such a number. The different worlds can finally coexist in unreality.

Because the probabilities that the possible worlds are assigned sum to 1, a new reality emerges after this *synthetic* “1,” which is subsequently recognized (identified) as the “real” world in which the possible worlds are projected and weighted probabilistically. We are led to believe that the transition between the possible and the real takes place between this synthetic reality (where the possible worlds are embedded) and the reality of the one possible world that will become realized. The synthetic real world, however, is no less a fabrication than the possible worlds. Its only purpose is to situate and measure the probability of the possible worlds. Both the thought of the possible and the thought of the synthetic real (in other words, the thought of the transition or the process whose numerical expression is probability) come *after* the illegitimate exchange of the contingency of the one and real world. This contingency should have never been exchanged, at least not in this way. Identity should have never preceded difference. Contingency should have never been expressed in possible states. Time should be measured and punctuated by a transition other than probability.

Absolute contingency vs. delimited possibility

Possibility is fabricated after reality and this is why all prediction models and all probabilistic forecasts are vulnerable to contingency, which is the other name of reality. Possibility is *derived* from reality; it follows reality (rather than the opposite) and for this reason is always weaker and poorer than reality. Reality is altogether more incredible than any imaginable fiction. We always start from reality and all we think we have to do in order to describe another reality, or another possible world, is to vary a few items or states of affairs that we have conceptually iso-

lated in actual reality. This is the extent to which we imagine something we call the “possible.” Possible worlds are thus always subordinated to the real world. We trick ourselves into thinking that the future, or the unpredictable, is no more serious a game than the random realization of one among the possible worlds that we have so unimaginatively sampled.

The truth is, however, that the future will massively change the reality from which we have extracted the possible worlds in the first place. Genuine events are those that redefine possibility. They do so simply because they change reality and because possibility, as I have said, is derived from reality. As the saying goes, real events “create” their own possibilities. No wonder that the possible path leading to the event is always reconstructed after the event. The backward narrative, which Nassim Taleb recognizes as the third characteristic of the Black Swan,⁵ in fact contains the first two: the high improbability and the large impact. When the true event is recognized as being a change of the whole range of imaginable possibilities (i.e., as a complete departure from each and every variation of the states of affairs that our thought was previously able to distinguish in the reality we know), it is only normal that it should be highly improbable and have a large impact. It is a shift of the whole space of possibilities on which probability was previously defined. It is not even accurate to call the event highly improbable or to think that it was assigned zero probability in the previous set of possibilities. The event wasn’t even part of that set and its probability wasn’t even defined.

To this, the supporter of prediction models will object that he is perfectly willing to grant that, in matters concerning the unknown future, anything can happen indeed, even beyond the scope of possibility, yet he cannot see what the alternative to possibility might be. For all the weakness and derivative character of possibility, at least it offers to our thought a grasp on the future, if very limited. Wouldn’t the definition of contingency as change beyond the imaginable otherwise condemn us to total impotence? If we are ourselves setting this insurmountable limit, how are we ever to cross it and say something relevant about the future? Absolute contingency is

such a devastating idea relative to the pedestrian notion of possibility that it cannot qualify as a criticism of possibility.

When distinct possibilities are identified, the risk, of course, is that some unidentified possibility emerges and devastates our schema or representation, nay, the whole extant order of possibility as a vision of the world. It is almost as if identification created itself the eventuality of its ruin. The inside calls automatically for an outside. However, the radically different or the other-worldly is not the only liability of identification. There might be reasons, pertaining to the actual and perfectly familiar world, not to commit the act of identification and of delimitation of states. (I’ll give you a hint: imagine a totally free world – a free market – without boundaries and states.) As I have said, contingency is not only the vengeance that the real exercises against the possible by subsequently shaking the whole domain of possibility. Before we come to this extremity, contingency can be seen as the edict *not* to identify the world or identify its possibilities in the first place. It would be a stronger criticism of possibility if we were able to show that the real world shouldn’t be separated into distinct and delimited possible states to begin with than to expect reality always to exceed the range of possibilities after the range is fixed and to bring about a future contingency – that is, an unforeseen possibility. Before showing that the integral calculus of possibility always fails, it might be more efficient to show that even its differential calculus is not available.

In other words, if contingency is what eludes possibility and is independent of it, it should be possible to translate contingency without the mediation of possibility. (Maybe such a translation is already operative in some parts of our world and would be recognizable if only possibility withdrew its screen.) That the possible states of the world are always vulnerable to events that exceed their range should be the *consequence* of the fact that absolute contingency is independent of possibility, not the definition of contingency. What we wish to criticize in possibility is not necessarily the limitation of its scope but its mechanism.

Contingency is even independent of time



(where time is understood as that which eventually brings things into existence) and the sense should be the same in which an existent and perfectly settled thing is said to be contingent (i.e., it could have been different) and in which a future, nonexistent thing, is said to be contingent – that is to say, not determined by necessity and not even certain to come about. The reality and the contingency of a future thing should be thought ahead of time, so to speak, in the same medium as the thought of the contingency of an actual thing. The thought of contingency comes before the thought of existence. It is not possibility that produces the reality of a thing (it is rather the opposite); therefore, the contingency of a future real thing must be separated from the projection (the fiction) of future possibilities.

It is perfectly contingent that a thing exists or doesn't exist and, if it does, that it exists now or later, under this form or another. Therefore, the heuristics for thinking of the contingency of the future world independently of the thread of possibility would be to think of it as if it was already actualized *and then* to think that it could have been different or, for that matter, that it could have never existed.

Dynamics of possibility vs. dynamics of reality

The mechanism that we wish to criticize in possibility is the *transition* between the possible and the real. This is where all the fabrication and the artifice are condensed. The transition is the consequence of the identification of states between which it is supposed to occur. Instead of dismissing the limitation of possibility by an extensional argument to the effect that contingency can never be covered by a total set of possible states, we should be able to dismiss the transition – or the differential and delimited character of the states – by a logical argument. What is it that is so incompatible with possibility in the *nature* – and, shall we say, in the sense – of contingency? What is it, in contingency, that is so different from possibility and irreducible to it, independently of the argument of the total of possibilities?

Possibility is said to become *realized* while contingency is *real*. This is perhaps the main difference in nature between the two. Realization

implies a transition between multiple possibilities and a reality that is singled out transcendently, while the reality of contingency is transmitted univocally and immanently from the one and only real world to the next. The only problem is that we do not know how to stage the movement from real to real, or even whether there is such a movement to be staged externally. What could take care of the transition from one day to the next if no explicit (temporal) *process* is distinguished within the real? The figure of the tree imposes itself as the natural picture of evolution and process. We perceive motion much more easily and more naturally than we distinguish the details of a still image (this is even coded in our genes of former residents of the wilderness). This is why we like to track the motion of the symbol of reality, “1,” as it descends along the tree of possibilities. The tree mediates reality and processes it. It sets the stage for what Badiou calls (commenting on Deleuze) the “reciprocal play of beings” (play, even in the theatrical sense). It is very difficult, by contrast, to extract any thought or any mediation from the single – and, as it were, “static” – stroke of contingency.

One of the advantages of the dynamics of the realization of possibility is that it is a dynamics, precisely. It suggests that the actual world is the realization of one of the branches whose probability was less than 1 only yesterday. We think of today's world as issuing from yesterday, as a possibility that is freshly realized. The weight of reality remains, in a sense, entirely supported by yesterday. We think of today's world, when it comes into existence, as being no more than one of the branches emanating from yesterday's world. The truth is, however, that yesterday's world, together with the alternative branches rooted in it, are only a fabrication. They were entirely made up for the sole purpose of simulating the contingency of the present world and simulating the passage of time.

The real world is only envisaged as the *realization* of a possibility and this is why the symbol of reality, “1,” is as a matter of fact misplaced in such a picture. It is always situated one step back. Even though today's world is realized and yesterday's world is no longer real, at the back of our mind the symbol of reality remains focused

on yesterday's world because of the thought of the transition – because we picture “1” as being handed over to the present world by the previous world. In reality, however, yesterday's world was never real in such a picture. It was only meant to support the sum of probabilities of unreal possibilities.

Instead of conceiving of the present world as having acquired, only just, the symbol of reality by virtue of the realization of its possibility, what would the alternative dynamics be? What would the *real* dynamics be, whereby the “1” of reality does not become attached to the present world as the result of an acquisition or as the conclusion of a transition, but is firmly attached to it from the beginning? What would the dynamics be, whereby the one and only real world is followed by the one and only real world, instead of multiple possibilities being followed by a realization? What would the dynamics be, where the focus is put on the present real world, immanently enfolding the transition to the next, not on the faked past of which the present world is only supposed to realize a possibility? What would the dynamics be, in other words, where the hierarchy of the possible and the real is reinstated in the right order and the possible is thought to follow from the real – if it must be thought at all – instead of preceding the real? How, indeed, to turn, from the backward movement of probability to the forward movement of contingency?

Writing contingency

We know that it could rain, tomorrow, or, equally, that there could be sunshine. Tomorrow's world is indeed contingent. Why do we have to represent this as a fork, with two different worlds being assigned two probabilities that only reflect the mixture of two incompatible realities – the reality of rain and the reality of sunshine? What representation of tomorrow's contingency can we presently have, other than two states of the world, or two possible worlds? How to represent tomorrow as a single world, only a contingent one, and still manage to connect it with the present world through a *process*, only without a transition?

My rather bold claim is that there is no such thing as possible states and that writing is the

only thing there is (the only thing that can mediate contingency). In reality (or in writing), we don't have possibilities. Possibilities are not real; therefore, they cannot be had. What we have are contingent claims. What exists and is material today are two *contingent claims*, one of which pays 1 tomorrow in case of rain and 0 otherwise, and the second of which pays 1 tomorrow in case of sunshine and 0 otherwise. Both contingent claims belong to the actual present world, and they will both belong to the one and only world that will take place tomorrow. What separates them and distinguishes them are not states of the world, or different worlds, but different formulas. It is true that the formula is differential and that it consists of two clauses: "If it rains then..., else if it is sunny then...." However, it is a single formula; it doesn't divide the contingent claim into two realities. The whole question is to recede from the expiration of the contingent claim (or the time when the formula is opened and the payoff triggered) without decomposing the single formula – what I call the "stroke of contingency" – into separate possibilities. We have to *translate* the stroke (literally, to move it in time and space) into something else – a medium as real as contingency, something altogether faster and firmer than the unreal intermediation of possibility.

The contingent claim is this single (differential) formula. It is this strike. This is its definition. Writing is an etching, marked on an otherwise indifferent surface. Writing points to a difference. Writing is difference. If there were nothing different and nothing remarkable, there would be nothing to mark and nothing to write. In order to engrave the difference, writing needs a material sheet – something Roland Barthes calls the "subjective matter."⁶ "Subjective" literally means *sous-jacent* in French, the English translation of which is *underlying*. This real and material underlying, on which writing is engraved, is what replaces the whole theatre of possibility and the play of its underlying states. Instead of thinking of two different worlds that can only coexist in unreality and instead of fabricating a synthetic world whose only purpose is to "temporarily" store the symbol of reality, "1," before it hands it over to the world that will be realized,

we materialize the thought of the two worlds into a contingency; we make the two incompatible "worlds" coexist *in reality* as different writings (no longer as different worlds), materially engraved on top of two real contingent claims.

I insist that we really withdraw the states; we are left with contingency pure and simple that is no longer derivative on possibility. The written formula collects as one writing the two branches of the alternative which are incompatible in actual reality; this is feasible by the alchemy of writing. We tend to forget what writing can do!

Instead of the fork of possibilities leading us to the world that will be realized, it is writing that now acts as our guide. The rainy day and the sunny day, when they were thought of as two unrealities just in order that they could coexist the previous day and that the transition could be defined, were eventually discriminated by the transition – exactly as originally planned and contrived. Realization of one of the branches acted, then, as our guide. This, however, was

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only a mental operation. The acquisition was not real because the two "days" did not exist the day before. How, indeed, could an unreal thing suddenly jump into reality? In what inclusive reality could such a movement ever take place?

We know that it either rains, the next day, or that it is sunny, and the real world in question "acquires" the symbol of reality anyway – a symbol it has in reality never parted company with. We know tomorrow to be real, yet to be contingent. The whole question is how to *really* step back in time, to the day that is really the previous day and not the fictive day whose only purpose is to support the possible branches. How to step

back from the contingency of the rainy/sunny day while preserving the full contingency of the day and not filtering it into the two states that we have identified beforehand? How to allow for the fact that, although our only interest in the future may be the weather forecast, the previous day in no way reduces the scope of contingency or even breaks down contingency into states? For all we know, a meteor can wipe out the surface of the Earth the next day, or the crust of the Earth can destabilize, or the laws of gravity can change, etc.

How to make it so that the identifiers of the event of rain and of the event of sun become an integral part of the reality of the previous day and do not just stand there as possible states, as unrealities? How, indeed, to *have* the future and the future contingency without fictionalizing the future? In a word, what is the *real* thread of the future? Isn't the whole thought of possibility (and the corresponding metaphysics) just the consequence of the fact that, for the reason that we happen to think that the actual is the only

real, we find it hard to believe that the future is real? Do we have to know and identify the future in order to think it is real? It is not even sure that we have to identify the present world! For all we know, the thought of contingency as derivative on being and on state – when contingency has to be thought as absolute and underived – might be entirely due to the unnecessary, yet irresistible, identification of the present world. What if we found that the proper medium of future contingency (and not of possibility) had nothing to do with knowledge or even forecast? What if the future was *written* – which does not mean that it is settled, of course – and writing was the

medium that is impermeable to knowledge or prediction?

If a process other than the realization of possibilities has to connect us with the future, a process that moves from real to real and not from possible to real, might not this process be the writing and subsequently the translation of contingency, as such unadulterated by possibility or state? And would we be surprised, as a result, that the future should be real? The rather strange result that we seem to be obtaining here (a result, the ramifications of which remain to be explored in full) is that the notion of state and identification of states is what's standing in the way of the reality of the future.

Genesis of price

Writing is our guide and the contingent claim knows how to behave in case of rain or in case of sun, the next day, based on the formula written over it. The state of possibility also knows how to behave. It acquires the symbol or reality if it is realized, making it look as if probability had fulfilled its mission of mediator between the multiple and the one. However, probability doesn't require writing. The whole operation is immaterial. By contrast, the reason why the contingent claim is written is that it will be cashed against something. The contingent claim bearing the formula of rain will be cashed against "1" in case of rain – a symbol of reality of a new kind, material and not abstract, yet every bit as ideal as a number.

What is this "cash," what is this "money"? What does it mean, "to pay"? The payoff is also a realization, it is also an acquisition; however, it transits between real and real, not between possible and real. Recall that the *numeraire* (or medium) in which probability was expressed was not homogeneous. While "1" was the symbol of reality, p was the symbol of unreality. Just as probability was associated with possibility and possible states, only suffered from the heterogeneity between the possible and the real (as well as from the false backward movement being the only way to apply probability to the delimited states that we had identified within the real and falsely projected backward in order to stage the possible), *price* is associated with the *written*

contingent claim, where, by contrast, writing is always real.

From the moment writing was invented in its capacity to mediate contingency – to mediate it materially, not conceptually (i.e., to present us today with a formula, a mark, a seal that can be opened and interpreted later in the light of the contingent world that will have come to be actualized) – what this materialization meant was cash and money. Better, what the very material that writing was made of really stood for and really stood to be exchanged against – a material such that the seal it bore would be opened later, yet a material that existed today and was real today – was money, quantified by price. Just as the thought of possibility prompts the thought of probability, the material of written contingency prompts the material of price. (This hints to the proposition, which I advance in the book, that the market and the price are the *materialization* of thought; therefore, they are the way in which thought can get outside itself.) We write contingency in order to mediate it really and materially – that is, without the false division of states – and the sheet on which it is written does not then stand to acquire just *any* symbol of reality. It stands to be cashed out for reality, through money.

Writing, price, and money are all being implicitly and jointly defined here, through the one and only decision to find a material alternative to possibility. What is a contingent claim? It is something that replaces the transition between the possible and the real with a written formula that is continuously material and continuously real, a formula that *traverses* the real. What is money? It is the materialization of the abstract scale of probability, something that was invented concomitantly with the contingent claim in order to act as the reality against which the contingent claim is cashed, both at its maturity and before. What is the market (or the exchange)? It is the *translation* of the contingent claim, literally the medium through which the contingent claim is moved from its maturity – from the future time when the world in question is actualized and is no longer future yet is still massively contingent – to the present time, *without this backward movement opening the exten-*

sional space of possibility.

While the contingent claim was written with the purpose of paying out and being exchanged against cash at its maturity, now that it is written, the material it is written on still finds cash to be exchanged against before the maturity. It finds it in the market. The written formula sets the price of the contingent claim at its maturity, a price that will vary depending on the then actualized contingent world. (This is the time when the seal is opened and the formula opens up its branches. This is the *characteristic* of writing.) And the formula finds a price, prior to maturity, by virtue of its holding as a formula and of its capacity of being held (by someone). This is the definition of the market.

Think of the market as the missing piece that makes all the pieces fit together and makes the construction (literally) fall into place. Possibility could find the symbol of reality only at the time when it was realized and, for this reason, the only thing that it could find prior to its realization was probability. Similarly, we have to think of price as the only thing that the contingent claim can find before it expires and its seal is open, except that price is real, while probability is not. This is how price is implicitly defined.

Mathematics of price

When "1" is the probabilistic symbol of reality, it can only be assigned to two (or more) mutually exclusive worlds by virtue of a fiction. We assign "1" to a possible world in the fiction of its future realization and we step back to the present to get its probability. We assign "1" to the other world in a different future, and we step back to the present to (improperly) mix its probability with the first. By contrast, when "1" is the price, it attaches to the two (or more) contingent claims at once without contradiction, because it is simply written on them. The two contingent claims, "RAIN tomorrow" and "SUN tomorrow," really coexist in the actual real world. That they should pay out "1" in case of rain or sun, respectively, is a real condition presently written on each one of them. Price is the transposition in writing, therefore in the material real, of the unreal assignment of a possible reality. Once the move is decided to replace unreal possible states with

real contingent claims, price is what replaces, in the real, the prop of unreality that we had added to the real in order to stage the possible.

In the present world (also called *spot*), whose time is prior to the expiration date of the contingent claims, the price of the present weather condition is 1 and the prices of “RAIN tomorrow” and “SUN tomorrow” are less than 1 and add up to 1, exactly like probability. Indeed, anyone buying today the combination of “RAIN tomorrow” and “SUN tomorrow” will be guaranteed to receive 1 tomorrow, no matter the outcome.⁷ Crucially, “RAIN” and “SUN” are never conceived as possible states in this arithmetic. They really exist today, only at the state of writings marked over the sheets of “RAIN tomorrow” and “SUN tomorrow.” If the real world turns out to be rainy tomorrow, the price of “RAIN tomorrow” will be 1 and the price of “SUN tomorrow” will be 0, as prescribed. Crucially, the rainy day is now real yet is contingent. It never was possible. At no point was there a transition from the possible to the real.

The exchange place

We are distracted from perceiving the purely logical nature of price by the impure and mixed image of the market. We think that the price is the result of a very complex auction mechanism, the intractable consensus of a whole crowd; when I say that the crowd is incidental. It is not so much the number of people that is the issue as it is the place where, as I have said, all the pieces fit together or the place that the contingent claim falls into when the thought of possibility and its underlying states is withdrawn. Consider that the assignment of probability has never involved a number of people. The assignment (or rather, attachment) of price to the contingent claim shouldn't require a number of people either.

In fact, what the category of price essentially involves, namely *place*, is precisely what probability lacks. The question of probability has always been unsettled between objective probability and subjective probability. I do not interpret this dichotomy in terms of the human being or the human mind but in terms of *place*. My explanation of the unsettlement of probability is simply that probability is unable to find its place. Its definition keeps oscillating between inside and outside,

between sticking inside somebody's mind and sticking outside, on the things themselves.

Probability is unreal, and for this reason it has a hard time residing in external reality. And when it is invited inside the mind, the question becomes even more mysterious of how this notion, whose first intention is metaphysical and not psychological, is ever going to apply to the external world or to physics. Simply, probability could not take advantage of the *materialization* of thought that only writing would later bring about.

Writing is neither subjective nor objective. It doesn't trade on the interiority of relation that is typical of the delimitation of possibility and of the nice reciprocal play of its states. It is not a question of *who* has written the contingent claim and *who* will attach a price to it: what transcendent subject. From the moment it is written, it finds its exchange place because what it stands for, as written, is cash and because what can stand for it and stand in its place when we realize that it is *still* written (and by this I mean that it “remains” written even before its maturity) is cash.

The market is the transposition of the “place” where we think probability gets assigned, corresponding with the transposition of possibility into the contingent claim – except that the hypothetical “place” where probability is assigned is precisely unsettled and ill-defined and the writing of the contingent is what settles it finally. As I have said, the whole puzzle is resolved and all the pieces just fit together, including the puzzle of probability. As it is resolved, however, probability is dissolved. Finding its reality (i.e., replacing it by the market of contingent claims) is at the same time its disappearance as unreality.

The place of the contingent claim is nobody's place in particular. It falls to no subject to assign a price to the contingent claim or to reflect it in his mind. Nor is its place an external theatre where multiple subjects gather in order to exchange the contingent claim and assign a price to it as if from outside, as a *result* of their transactions. The contingent claim is *shot through* with the exchange and the exchange is written all over it. To materially write it, in place of laying it over the underlying states, is to exchange it already and to let it fall in the exchange place at the same time as, and insofar as, the states are withdrawn. The place is a

place where anybody can be and where everyone is contingent. Statistically, this can only be the place of many and this is why we usually find more than one trader in the market.

The contingent claim doesn't meet with possibility or probability prior to its maturity. It was written precisely to meet only with reality. What reality it meets is the market and the exchange (we will see below in what dramatic fashion). The only way to affirm that the future world is real yet is contingent and to say so before the world is actualized and without falling into possibility in the meantime is to say it in the market. The market is the only way to preserve the reality of contingency ahead of time. We say that the world is contingent before it is actualized; we say it in reality, not in possibility. We anticipate the actuality of the world faster than any tree of possibility, by an instant transmission through the one and only medium of contingency. This medium is the market.

One has to understand that, from the moment that one has marked contingency over a material sheet and left it to writing as the only guide and no longer to the underlying states of possibility, this leaves the exchange place, or the market-place, as only a “massive” (or dark, or invisible – as a matter of fact, it is called a *pit*) medium where the transition without the states can be accomplished. The market is the immanent subject that guides contingency and translates it into the price.

The derivatives market and the case for probability

We said that the limitation of possibility in front of absolute contingency (whose characteristic is to shake the whole range of possibilities and to create possibilities that were previously unheard of) was not the only reason why writing contingent claims was preferable to conceiving of underlying states or the only reason why the forward and massive movement of price was more robust and more real than the backward and sophisticated minuet of probability. The transition from the possible to the real – a necessary consequence of the artificial identification of underlying states and their delimitation – was, we said, the real target of our criticism and the reason why we



went looking for an alternative medium of contingency, or a guide which would overstep the intermediation of states. We found writing and its immediate translation (the market) as the way of making present the reality of a contingent future thing before that thing became actualized. This would prevent us from projecting it into unreal possibilities and would take place at infinitely higher speed than the irresistible thought of possibility. The substitution of possibility by writing was such that we would now think of the future contingent thing no longer as a possible thing but as a real thing whose actuality we would anticipate (or, at any rate, would be indifferent to its contingency) and of which we could say – in much the same way as we would say it of an actual or even past thing – that it could have been different.

As we have indicated, the market as translator of contingency would consist of a medium, which is alternative to chronological time and to possibility, such that there would be no difference ultimately between the contingency of an actual thing (“It is the way it is but it could have been different”) and the contingency of a future thing. The whole idea would thus be to be able to think that a thing *can be* (different) before we think that it *is*, or to think its contingency before we think its identity or its state. We think the contingency of an actual thing before we think its *actual* state and similarly we think the contingency of a nonactual (i.e., future, or simply inexistent) thing before we think its *possible* state. When contingency is unshackled from the state, past and future contingency become the same.

The risk, of course, is that the defender of probability might object that all these subtleties and distinctions are only metaphysical. He would argue that, apart from the obvious shortcoming of the notion of possibility, which is that the fixed range of possibilities will always be vulnerable to contingencies not previously listed, he sees no difference between the probabilistic transition and the pricing transition. Take the roulette wheel, for instance. What is the difference between speaking of the probability of the numbers of a roulette wheel and speaking of the price of contingent claims that would pay out in case the corresponding numbers are drawn?

Surely, a fire can break out inside the casino

or war can be declared outside (in a spin of the wheel of destiny). Surely, we can argue that these contingencies would ruin the delicate mesh of probability, whereas they would leave unaffected the resilient fabric of the market. They were not part of the universe of possibilities underlying the probability calculus of the roulette wheel, and for this reason they completely upset it, whereas they leave unaffected the market of the corresponding contingent claims because the market doesn’t care about underlying states. It knows only the immanence of price and exchange.

The defender of probability would reply, however, that such events are so overwhelming that it doesn’t make sense to list their possibility in the universe underlying the game of roulette. Once it is agreed to exclude from the range of possibilities those that are so heterogeneous that they change the whole game and change the whole subject, won’t the transition between the possible and the real then perfectly mimic, and, for that matter, advantageously analyze and represent, the so-called massive transition between the contingent real and the following contingent real? Isn’t the possible, in the end, just a *model* – that is, a useful simplification and elucidation – of the contingent?

As a matter of fact, doesn’t the derivatives market itself encourage such a folding back of contingency on the homogeneous scale of possibility? Aren’t prices precisely the way of enlisting contingencies back into the ranks of numbers and quantifiable repartitions, in such a fashion as to precisely apply probability theory to the whole field? When contingent claims admit, as their payoffs, conditions that pertain to the traded prices of other contingent claims, doesn’t the field then close itself to external contingencies, or, to put it differently, doesn’t the market consequently internalize any such external contingency? Isn’t the derivatives market precisely the ultimate victory of the possible over the real, the victory of the abstract over the concrete and material? Hasn’t the derivatives market precisely been reproached with turning the hard economical facts into abstract numbers and with turning the whole economy into a “casino” where players no longer deal with realities but constantly speculate on pure possibilities?

The states that underlie a given derivative are

the prices of another asset, known as the derivative’s underlying. This completely determines the range of relevant possibilities. What new and unforeseen possibilities could indeed be relevant to the derivative price and change its world in ways not previously represented by the states of the world of its underlying? When all that matters is the series of successive underlying prices, doesn’t it become irresistible to represent the series in a stochastic process? Doesn’t the fork of possibilities become irresistible again?

Translating contingency outside possibility

I wish to argue to the contrary. When the states underlying a contingent claim are the prices of another contingent claim, the effect of the underlying being tradable on the contrary crushes the fork of possibility back into massive contingency.

Recall that the main virtue of marking the contingent payoff over the material sheet of a contingent claim is to withdraw any distinction or delimitation of underlying *states* that the written formula naturally suggests and to keep only, on the surface, the massive and undivided contingency that is not reducible to states and could be anything whatsoever. Writing contingency over a material sheet was, as we recall, the way to recede from the expiration date – or from the day whose contingency we ultimately wish to test and “evaluate” today – without this backward movement imposing on us the necessity of a transition between states – that is, a transition between the possible and the real.

On expiration date, contingency is “massive and undivided” because the target world is now actual and no longer possible, yet it still incorporates the thought that it could have been different. Because the world now exists in its actual state, we do not need to identify the other possible worlds (or states) that it could have been in order to affirm its contingency and to admit that the world could have been different. We probably don’t even need to conceive of the other possible worlds. Strangely, the reality of the actual world is capable of transmitting the thought of its contingency without raising the possibility of other worlds. The thought of contingency is capable of traversing the wall of actuality unharmed,

whereas possibility crashes, of course, against such a wall.

To repeat, an actual thing is still contingent but can no longer be possible. Better, contingency is inherent in its reality; its actuality is only one aspect of its reality; therefore, it doesn't exhaust its reality. That we should be able to perfectly perceive the contingency of an actual thing or an actual world without necessarily fantasizing the alternative possibilities is the indication that, deep down, we think of contingency *before* (not after) actuality and existence. Contingency is the primary ontology. We instinctively think that things *can be*, before we think that they *are*.

When the world lacks actuality, however, when the world is still future, we confuse its lack of actuality with a lack of reality and we tend to think of a future contingent world under the category of possibility. What we miss is the materialization, before the maturity date, of the very medium that had allowed us "then" – that is, at the time when the world "was" actual – to think of its contingency without raising the question of possibility. Although the world is not yet actualized and possibility can prosper in the meantime, we wish to keep (or somehow to hold in advance) the "wall" that had blocked possibility at maturity. We wish to retain that part of the reality of the future actual world that "was" distinct from its actuality and incompatible with possibility. As I have said, we wish to recede to a date prior to maturity while keeping unchanged the thought of contingency that "was" independent both of possibility and actuality, literally translating it.

I hold that this translation is possible thanks to writing. Writing *can* support the reality of contingency (which we instinctively recognize to be independent of possibility at maturity date – i.e., at the time when possibility is precisely dead) without resurrecting possibility. Writing is the reality, or the material, that we need to get hold of, ahead of time and before the maturity date – provided it is supplemented by its other face, or rather, by its only remaining face (now that the underlying states of possibility have been withdrawn and that writing is no longer thought to be derivative on those states), namely, the exchange.

What is at stake, metaphysically, in the capacity to recede to a prior date without opening

possibility – a capacity that only writing, I claim, can fill and fulfill –, is that such a backward movement shall not take place in chronological time. The price process is not a temporal process. Although an interval of time physically separates the present price or the present market of the contingent claim from its maturity date, the rather amazing claim here is that the succession of prices that fills up this interval does not unfold in time. The prices do not follow each other in time; they are not generated by a random generator; they successively repeat the *whole* genesis of price.

The accident of chronological time

Price, we said, becomes the substitute of probability as soon as we manage to replace the states of possibility with the written contingent claims. However, the objection now is that the states underlying our contingent claim have not been withdrawn. As a matter of fact, they are still alive and trading because they are precisely the prices of another contingent claim. So, again, apart from the nice metaphysical elaboration to the effect that the exchange is the proper medium of the reality of contingency that is faster than possibility and that does not require its intermediation, could things not be behaving, after all, as if the prices of the underlying were really underlying the values of the contingent claim and as if those values were just computed under the corresponding probability and mathematical expectation? Now that it is accepted that prices are the translation of contingency and that the market is the reality of the future contingent world ahead of its actualization, and since prices, both of the underlying and the contingent claim, are now present everywhere, can't their joint evolution be modeled again by probability and stochastic process? Now that we all understand what prices are really made of and what material contingency they transmit (as opposed to ethereal possibility) and since price movements are really all that can happen in a market, isn't possibility likely to take over again, if only for ease of representation and for numerical tractability? In other words, isn't a temporal process likely to fill up again the stretch of time separating the present price of the contingent claim from its maturity?

We have to take this question seriously because

the risk, here, is that representation might take over again and that the *genesis* of price might be forgotten – that is, the whole sequence that went from the marking of contingency over a material yet free-floating sheet (instead of chaining it to underlying states) to the meeting of this written contingency with price in the reality of the market (instead of meeting with probability in the unreality of possibility). Since we are separated from maturity by a stretch of time and we do not yet have the final clause stating that the contingent claim pays something but could have paid something else, since all we have to hold on to is the underlying state (which is alive and trading), doesn't the tree of possibilities, leading from the present underlying state to the state that will become actualized at maturity, seem irresistible again? How, indeed, to repeat the thought that contingency is independent of the underlying state now that the latter seems so pressing? How to repeat the erasure of the underlying state under the materiality of writing? How to repeat that price is not the result of probability but the product of an unsettlement or a tension between the different branches of the contingent payoff – that is, the product of the difference that the writing of the contingent claim materializes, a surface tension that has no other way to resolve itself but in the market?

The rather strange observation is that only at maturity is the contingent claim really derivative on its underlying because its price is then settled and rigorously equal to that function of the underlying called the payoff, yet it is at maturity that the underlying state is no longer a *possible* state (it is actualized) and hence it is easiest to withdraw it from underneath the contingent payoff and subsequently to claim that contingency is absolute and no longer derivative on that state. In other words, although the contingent claim is perfectly determined by its underlying at maturity, the fact that the underlying is now an actuality and no longer a possibility and that contingency is still thinkable (and as matter of fact pressing) enables it to escape all the better from the shackles of possibility and to express itself absolutely.

This is just the expression of a very interesting property of maturity (which is in fact just a coincidence), a property which allows us to think



that the contingent claim is contingent because it could have paid out something different in actual fact, yet without this difference being identified as a branch of the tree of possibility or as driven by possibility. It is exactly at this point of liberation of contingency (from the tree of possibility) that we wish to place the exchange and that we wish to say that the contingency without the underlying states falls into the exchange. This is precisely the genesis of price; however, price, if left in that state, would be stillborn. Indeed, the “side-effect” of maturity is that the exchange is then automatical-

It is dynamic replication that can reproduce or repeat the maturity. It can repeat both the determination of the contingent claim at maturity and the liberation of its contingency from possibility. And it can do so before maturity, with time left in order not to terminate the exchange

ly rendered useless, for the contingent claim precisely expires at maturity and its price is settled.

To repeat, the temporal paradox of the exchange is that the exchange best expresses itself at maturity, for the reason that it is only at maturity that we know how to liberate contingency from the tree of possibility and how to let it fall into the exchange, and yet the fall into the exchange is almost immediately blocked by time – by the “accidental” fact that maturity is *also* the time of expiration. The exchange is best defined at maturity and its idea finds its best start at maturity, yet the exchange can only take place prior to maturity. Our whole point is then to try to step back in time in order to separate the exchange from the maturity that terminates it, yet without this stepping back reawakening the expired possibility.⁸

It so happens that the tradability of the underlying avails us precisely of such a marvel. Granted, we find ourselves standing before maturity and

we trust that we are surrounded by the reality of the market and not by the unreality of possibility – by prices not by probabilities. But our problem is that we can never be sure. We cannot guarantee that the tree of possibilities is not simultaneously unfolding under our feet or that prices are not independently following probability, and this fact is what makes us long for the expiration date. What we need is the repetition of the expiration date, the repetition of the genesis of price or of the capacity of saying, “It is too late for possibility, yet the contingent claim could have paid differently.”⁹

In a way, we want the contingent claim to be exactly determined by its underlying, just like it “was” at maturity (yet we want possibility dead), in order precisely to liberate contingency from possibility.

The virtues of dynamic replication

It is *dynamic replication* that can reproduce or repeat the maturity. It can repeat both the determination of the contingent claim at maturity and the liberation of its contingency from possibility. And it can do so *before* maturity, with time left in order not to terminate the exchange.

What we have always abhorred in possibility is, as you recall, its ill-adaptation to the problem of time. The reason why we have found that the time of expiration is best suited to separate contingency from possibility is that possibility could then no longer mix its improper (and unstable) sense of time with the eternal (and robust) sense of time of contingency (eternal, in the sense that it is inde-

pendent of time). Because of its tendency to fill up time with states that it will have only partially and arbitrarily identified in actuality and only artificially projected backwards in time, possibility interferes with the real passage of time – that is, with real contingency – and stops us from conceiving a future contingent thing other than as a possible state among other different states. Conversely, possibility projects forward only those states that it has sampled in actuality, or fabricated via unimaginative extrapolation; therefore, it is vulnerable to the emergence of new and previously unidentified and unimaginable states. In a word, possibility mixes up the time of radical change with the clock of the stochastic process.

Now, dynamic replication in the market precisely separates the automatic and fabricated time of possibility from the real passage of time. Indeed, if all that separates a contingent claim from its maturity, from the point of view of fabricated time, is a tree of possibilities, then, no matter how complex the tree or the contingent claim, a dynamic replication strategy that we know we could follow along the tree will make it so that the price of the contingent claim shall be a deterministic function of the prices of its underlying and of the other contingent claims that we would use in the replication strategy. It is as if the contingent claim had expired now and that all possibilities had been exhausted for it (exactly like at maturity).

In reality, however, there is still time separating us from the actual maturity. It is only the time of possibility (measured by the stochastic process) that has been abolished by dynamic replication. As for the time that is “left” (spanning the exact whole life of the contingent claim, mind you), it can only, therefore, be occupied by the exchange. In a word, the real effect of dynamic replication is to liberate the contingent claim from the underlying possibilities, as if writing its payoff was being repeated and the underlying states were accordingly being withdrawn, while, at the same time, preserving an expanse of real (not possible) time in which the exchange, and the exchange alone, can take place.

In this way, dynamic replication keeps open the place of the exchange; it reaffirms the market at every point in which possibility was threatening to take over contingency and probability was

threatening to take over price. Just as the withdrawal of the underlying states at maturity left no place other than the exchange place for the contingent claim to fall into, just so the rewriting of the contingent claim ahead of maturity – a rewriting whose other name is dynamic replication – opens the place where the market-maker in charge of dynamic replication can stand. Only in theory does dynamic replication render the contingent claims redundant. In the market, provided that the market-maker is affirmed, dynamic replication on the contrary dismisses possibility and abolishes theory.

Dynamic replication brings back the crossing and the repeated decision of *place* at a time when the tree of possibilities threatens to dissolve the place in the unreality of its algorithmic sequence. For this reason, the dynamic hedger, and the dynamic hedger alone, is entitled to receive the price of the contingent claim as a *price* and no longer as a value, and consequently to invert the valuation formula against the market price.

Implying volatility – the act of inverting the Black–Scholes–Merton (BSM) formula – only makes sense in the hands of the dynamic hedger.

The redefinition of volatility

If the contingent claim depended only on possibility, then, at maturity, when possibility is dead and buried, all sense of contingency would be extinguished too. Yet, contingency is indifferent to maturity (because of our inherent feeling that actuality doesn't exhaust the reality, or virtuality, of contingency), and it is precisely in the interstice between the extinction of possibility and the persistence of contingency that the concept of price emerges almost imperceptibly and ephemerally.

Price emerges transiently and cannot endure at maturity because no sooner does it emerge than it is settled and taken over by value (as payoff function). Thinking that the value could have been different, despite the fact that the possible state, driving value, is settled, is the narrow passage of thought in which price emerges. Price is in excess of value. It is the contingency of value, after value is determined.

The problem is that the price of the contingent claim is settled too at maturity and is then equal to value. Only prior to maturity are we able to

separate price from value, provided that value is settled and price is not. (The whole possibility of the market lies in this gap.) Recall that what settles the value (the extinction of possibility) is at the same time what enables us to withdraw the possible states. We withdraw the states because we don't need them but we still need the contingency (the "could have been different"). The problem is, however, that we only have the underlying state to hold on to before maturity and we don't have contingency. The only way to repeat the genesis of price and to reinstate the missing cross of contingency is the delta. This is the reason why we invert the formula of possibility and compute the delta ahead of maturity.

The formula of possibility is trivial at maturity. As a matter of fact, it is no longer a formula of *possibility but of actuality*. This is the formula that settles the value of the derivative and ultimately justifies its appellation of derivative. Going the way of the formula, at maturity, is going from the underlying state to the payoff function that derives from it. It is simply following a schedule: to this underlying state, let this derivative value correspond. This, as we know, is not the way of price. This decomposes the contingent claim into separate outcomes. The way of price is the reverse way. It seeks the *other* destination of the contingent claim and of its payoff formula, the one in which the material sheet it is written on is not decomposed into the different outcomes that correspond to the different states but turns its back on such decomposition and retains its undivided nature of a single sheet. This is when the straightforward representational logic that assigns a definite value to a definite underlying state is converted (and inverted) into the exchange. At the same time as the underlying states are withdrawn and the material sheet recovers its unity and materiality (its massive contingent character), the exchange replaces the logic of assignment and price replaces value, if only in a flash of thought (what I have called a "narrow passage").

Value is produced by actualization at maturity; it is dictated by the programmatic character of the payoff schedule. By contrast, price is produced by counter-actualization – by a reversal; it is produced by the retrieval of contingency or the insisting thought that, although the underlying

state and the derivative value are actually settled, the written and massive and material character of the contingent claim repeats that value is in fact unsettled and that it could have been different (i.e., it is a price). This last-minute unsettlement of value, which is productive of price and which reminds us in a flash that contingency does not need possibility, I offer to call *volatility*. Note that this definition of volatility does not occur in time; it is beyond the order of time. It occurs in the virtual.

Volatility is essentially attached to price insofar as contingency, which is differentiated from possibility at maturity and finds in price the translation of this differential character, expresses in volatility its "indeterminism." My claim is that volatility – according to my peculiar definition of the word – has nothing to do with probability theory and stochastic processes. It is not measurable on a quantitative scale. Notice that I am defining it at maturity precisely, when there is no time left in which the underlying process might still have quantitative volatility. It shouldn't be confused with the instantaneous volatility of the underlying either, for the latter is defined at the instant of maturity of course. As I have said, my qualitative definition of volatility falls beyond the whole category of possibility and the whole order of chronological time. This is volatility that is only due to the written and differential character of the contingent claim when the relative underlying states have been withdrawn, precisely. This is *absolute* volatility, corresponding to the "it is now too late, yet things could have been different." Since the world is actual and we no longer need to identify its possible states, since those possibilities are virtually anything whatsoever, the absolute volatility that is measured against them is "infinite" (in a nonquantitative sense of the word).

The reinterpretation of implied volatility

Prior to maturity, value is produced by possibility, or probability. When the derivative value is unsettled on account of the fact that chronological time separates us from its settlement date, the tree of possibility and probability theory are supposed to fill the gap. We talk of a stochastic process (e.g., Brownian motion) and we talk of its volatility.

However, the unsettlement in question is not the same as the unsettlement of contingency. Nor is the probabilistic volatility, which, in this instance, is productive of theoretical value, the same as the volatility that I have defined earlier to be productive of price. To recover price and the corresponding “absolute” volatility ahead of maturity, we need to repeat the following refrain: “Value is settled, yet it could have been different.” Just as we had inverted the trivial formula of possibility at maturity (in fact, the formula of actuality) in order to recover the intensive and undivided contingency, we have to invert the formula of possibility ahead of maturity in order to recover the price and absolute volatility.

This is my interpretation of the act of implying volatility. *Implied volatility* is the best witness that the market-maker never believes the tree of possibility to be productive of the price or that the price of the contingent claim can ever be the result of probability. While the fear was that, precisely because of the persistence of the states of the underlying, the tree of possibility might take over, it now appears that dynamic replication brings back the end of possibility and the corresponding unsettlement (i.e., the eternal return) of contingency. While the fear was that chronological time – which we definitely needed in order to let the exchange endure – might take over, it now appears that dynamic replication reintroduces the order that is beyond time at every point of time, through the act of implying volatility. Because of implied volatility, no stochastic process can ever start and consummate its preprogrammed time. Time is stopped at every point of time, so to speak, and a new a-temporal exchange (involving “infinite” volatility, just like at maturity) takes its place and reaffirms the exchange place.

Let us see how.

This is a situation, remember, where price has already been shown to take over probability and the exchange to take over the logic of assignment of value when the material writing of the contingent claim was reaffirmed independently of the underlying states, yet a situation where we fear that the redefinition of the underlying state as an underlying price (thus, derivatives markets) might bring back the tree of possibility and its

debilitating transitions between the possible and the real. This is a situation where the fate of the exchange is at stake, or, in other words, where the massive and undivided contingency, which we know can take place at maturity, despite and through the expiration of possibility and which subsequently generates price, is put to the test of time, or, more exactly, to the question of whether a backward movement in time could take place – and the exchange subsequently be given life – without the corresponding expanse of time becoming invaded by the fork of possibility.

The rather unexpected consequence of the redefinition of the underlying state as a price, however, is dynamic replication, and the consequence of dynamic replication is that the pricing formula (i.e., the probabilistic model) is precisely inverted against the market price in order to reproduce, prior to maturity, the death sentence of possibility: “It is too late, but it could have been different.” What dies at every instant, under dynamic replication, is, indeed, possibility (as the value of the contingent claim is completely determined according to the formula of possibility that the market-maker holds) but what rises and endures after the expiration of possibility (thus reproducing the absolute and eternally returning contingency) is the unsettlement of the dynamic hedger who repeats, “It is too late and I have now rebalanced my hedge; however, I could have rebalanced it differently.”

The slippage in dynamic hedging is precisely the origin of derivative value; however, this is only algorithmic slippage and the corresponding value is theoretical. How price replaces value in such a picture and how market replaces theory is by noting that the market-maker doesn’t hold the valuation formula in order to price the contingent claim but in order, precisely, to compute its dynamic hedge. The price is given by the market; however, what is not given and has to be reconstructed at every step is precisely the *theoretical dynamics*, or the very transition between the possible and the real, if we must insist on having one.

The reality of the market

We had feared that the underlying prices might have dragged us into the nets of the tree again; however, we now find that the consequence of maintain-

ing the belief in an underlying price *process* is that we should rely on the price of the contingent claim to reconstruct such a process, rather than relying on the stochastic process to compute the price of the contingent claim. We must imply volatility precisely because of time and because we are dynamic hedgers living in the market *and* in chronological time – that is to say, ahead of maturity.

By implying volatility, however, we turn it into a volatility smile; therefore, we need to upgrade the BSM model. By switching to a smile model, we need to imply its parameters in turn, what we call *calibration*. By calibrating the parameters of the smile model, we commit ourselves to recalibrating them and to turning them stochastic. As a consequence, the volatility smile model meets with its own, higher-order volatility smile. This means that it won’t be possible to calibrate the vanilla volatility smile and the price structure of higher-order exotics, typically barrier options, within the same instance of the smile model. A superior smile model (the smile model of the smile model) will be required in turn, in order to calibrate instantly both to the vanillas and to the barrier options. By calibrating such a superior model we commit ourselves to recalibrating it and to turning its parameters stochastic, and so on and so forth.

Presumably, chronological time will manage to slip between one instance of recalibration and the next. It accidentally slips in because the market-maker accidentally lives in time and because the market, when it is accidentally “slowed down” by chronological time, will not present at once the prices of the whole infinite sequence of exotics of every order. When it is observed that this accidental finitude of the sequence renders the smile model potentially undetermined (for instance, the unavailability of market prices of barrier options makes it impossible to discriminate between two smile models that agree upon the vanilla surface but disagree upon the smile dynamics – i.e., precisely upon the prices of barrier options), the conclusion is that the market-maker has no other choice, if he wishes to determine his model, but to publish himself the prices of the missing exotics.

What separates the present price of a contingent claim from its maturity is, therefore, an ever-inflating sequence of exotics of higher

and higher order for which the market-maker has to publish quotes. When we had feared that the medium of contingency, in which we had no choice but to inject chronological time, might be colonized by the tree of possibility and that the intervening contingencies might finally come down only to the probabilistic transitions of the sole underlying process, we now find a potentially infinite, nay, nontotalized, “chaos” standing in our way. This is not chaos in the sense of disorder but in the sense of its irreducibility to states and to division of states – a chaos whose other name is the market.

In fact, the chaos was with us from the beginning. It is just the expression of the massive and undivided contingency – of reality succeeding to reality. Only when we recede in the tree of possibility does the chaos become explicit. As a matter of fact, it is implicit in the stroke of contingency of the very first contingent claim we have considered. While it was perfectly possible to recede from the contingency of the winning number of the roulette wheel in a tree of possibilities that did not include each and every way in which the world could be different, and while the reason why this was possible was that, except for the roulette numbers, all other differences were either completely irrelevant or completely overwhelming for the game, the ways in which the market can be different, even when we are interested in a single contingent claim such as a vanilla option, involve all the other contingent claims. This is the one case where the delimitation of states of possibility, as soon as it tempts us, brings about the nontotal of contingencies as if from inside. It is, therefore, a necessity to criticize the identification of states, or the transition from the possible to the real, as the first, and perhaps most severe, defect of possibility. It is a local defect, harder to diagnose and more pernicious than the global defect which makes the whole range of possible states vulnerable to an external event.

The reality of the whole market worms its way into every attempt that possibility undertakes to precede the real. The market is its own source of contingency. In every single price, the whole market is enfolded, and volatility, according to our definition, is not approachable by a single

parameter, or a set of parameters, that we would infer through the calibration of a certain model. Absolute volatility enfolds at once the whole infinite sequence of recalibrations.

The existence of a market of contingent claims, where none should be redundant and all must independently trade, is a direct proof of the nonexistence of a random generator for the underlying. In other words, it is a direct proof of the nonexistence of states of the world. The whole metaphysical notion of possibility has to go away, together with probability.

Conclusion

Am I saying that we had to wait until the advent of the market of contingent claims, in ever more complex shapes, to finally discover that probability should have never existed? Couldn't we discover that from the beginning, from weaknesses inherent in probability and possibility themselves?

Bergson and Deleuze discovered these weaknesses and produced the corresponding criticism of possibility. No wonder the alternative metaphysics that I propose is exactly Bergsonian or Deleuzian. The contingent claims and their markets are just the vindication of Bergson and Deleuze. Here, probability theory really finds its limit and anyone disagreeing with Deleuze and Bergson is in trouble.

To repeat, this alternative to probability is not new (or overly sophisticated). It is as old as writing. Writing is even older than being (as someone like Derrida would say). Why are we confident that the market will always find a price for the contingent claim (no matter how complex)? We all have incomplete and very “shy” ideas about why it is so. We tend to think that the market participants all do some probability calculations, more or less accurate, more or less flawed, and that the market price is just the consensus. A consensus, or a price, is bound to emerge, we think.

Maybe so. But then, the important piece in such an argument is the last piece; namely, that a price always emerges in the market. It is not the probability calculations of the individuals that are important. As a matter of fact, they are all almost certainly wrong. On the other hand, it is meaningless to talk of the “probability calculation of the market.” The consensus is such an ugly mixture

ENDNOTES

1. See “The Blank Swan: Dan Tudball talks to Elie Ayache,” *Wilmott*, May 2010.
2. Meillassoux, Q. *After Finitude: An Essay on the Necessity of Contingency*. Trans. Brassier, R., Continuum, 2008.
3. Bergson, H. *Key Writings*, Ansell-Pearson, K. and Mullarkey, J. (eds), Continuum, 2002.
4. Baudrillard, J. *Impossible Exchange*. Trans. Turner, C., Verso, 2001.
5. Taleb, N.N. *The Black Swan: The Impact of the Highly Improbable*. Random House, 2007.
6. Barthes, R. *Œuvres Complètes*, vol. IV. Editions du Seuil, 2002.
7. Unless, of course, a meteor wipes out the Earth tomorrow or the laws of nature change. But this only means that the trader who has paid 1 to hold the combination of the two contingent claims would lose money the next day. It doesn't mean there was something wrong with the writing of the contingent claim and with their market (as there would have been with the delimitation of states of possibility). Losing money is a normal thing in the market. No particular law or logic is breached if prices do not evolve smoothly or according to plan. To repeat, we are chained to no particular tree of possibilities. Conversely, nothing tells us that the prices of the two contingent claims *really* have to add up to 1 today. In normal situations, they would. However, nobody can ultimately dictate the behavior of the immanent subject known as the market. For all we know, the market may “know” that a meteor will wipe out the Earth tomorrow.
8. Negating actuality while taking care not to fall in possibility, or unreality, can only take place in what Deleuze calls the *virtual*. According to Deleuze, and to Bergson before him, the virtual is the real that is not actual. For this reason, the medium of contingency in which we wish to withdraw in order to enable the exchange while keeping the sense of contingency that precisely exceeds the expiration of possibility and survives it, this medium, whose other name is the market and which falls completely outside possibility, is exactly the place of the virtual.
9. This repetition of the expiration date, which we need only insofar as possibility expires while contingency precisely survives, is the virtual.

that it certainly destroys all traceability to any initial probability calculation. My suggestion is, then, purely and simply, to boldly drop all reference to probability and probability calculation, and go straight from the contingent claim to its price. Yes, we are confident that the market always finds a price. All I am suggesting is that we see this as a simple statement, not as a complex one.